SECTION 17.7: STOKES' THEOREM

RECALL: Green's Theorem, Circulation Form: under suitable conditions: $\oint_C \vec{F} \cdot d\vec{r} = \iint_R (\nabla \times \vec{F}) \cdot \hat{\mathrm{k}}\, dA$

STOKES' THEOREM: Under suitable conditions: $\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{N}\, dS$

QUESTION: Can you explain how Stokes' Theorem reduces to Green's Theorem in a 2D setting?

EXAMPLE 1: Let $\vec{F}(x, y, z) = \langle 2x, z, 3y \rangle$ and S be the graph of $3x + 2y + z = 6$ which lies in the first octant.

1. Calculate the upward flux of $\nabla \times \vec{F}$ across S using a surface integral.

Ans: Upward flux of $(\nabla \times \vec{F})(x, y, z) = \langle 2, 0, 0 \rangle$ across S is 18.

2. Calculate the circulation along the boundary of S by evaluating the corresponding line integral.

   Compare your answer with #1.

Ans: Circulation totals to $4 + 18 + (-4) = 18$ ✓

EXAMPLE 2: Let $\vec{F}(x,y,z) = \langle 2x, z, 3y \rangle$. (Note: this is the same field from Example 1.)

Let S be described parametrically as $\vec{r}(u,v) = \langle 2\cos(u), 2\sin(u), v\rangle$, $0 \le u \le \frac{\pi}{2}$, $0 \le v \le 6$.

1. Calculate the 'rightward flux' of $\vec{F}(x,y,z) = \langle 2x, z, 3y \rangle$ across S using a surface integral.

   Use $\vec{N} = r_u \times r_v = \langle 2\cos(u), 2\sin(u), 0\rangle$.

   Ans: Flux of $(\nabla \times \vec{F})(x,y,z) = \langle 2,0,0\rangle$ across S is 24.

2. Calculate the circulation along the boundary of S by evaluating the corresponding line integral.

   Compare your answer with #1.

   Ans: Circulation totals to $-4 + 36 - 8 + 0 = 24$ ✓

HOMEWORK: 17.7: 5 - 41 every other odd.